Boolean Properties

Symbols

Logic

- AND - ∧ ⅄ • && &
- Complement - ∁
- NAND - ⊼
- NOR - ⊽
- NOT - ¬ ~ ˜ ! ¯
- OR - ∨ ⅂ + ||| ≥1
- XNOR or NXOR -
- XOR - ⊻ ⊕

Miscellaneous

- ⊢ - Provable
- O - Operator
- P or p - Proposition/Premise
- ∎ - End of proof
- ∀ - For all/any/each
- ∃ - There exists/is/are

Relationships

- ⇔ or ↔ or ≡ - Equivalence
- ⇒ or → or ⊃ - Implication-operator/Logical-consequence (implies)
- ≈ - Approximately/Almost equal to
- ≃ - Asymptotic equality
- ∵ - Because
- ⧐ - Between
- ← - Converse
- ⊮ - Does not prove
- ⊨ - Entails/Inference
- = - Equal/Same value
- ≡ - Identical/Logical-equivalence
- ‖ - Incomparability
- ⊢ - Inference
- ≅ - Isomorphism or congruence
- ◊ - It is possible that
- ≠ - Not equal to
- ≢ - Not identical to
- ⊭ - Not true
- ∝ - Proportional to
- ≣ - Strictly equivalent to
- : - Such that
- ∴ - Therefore
- ≐ - Variable approaches limit

Truth Values

- T or 1 - True/Truth
- F or 0 or ⊥ - False/Falsity
- I - Intermediate (neither true nor false, or both true and false)
- U - Undefined/Unassigned
- X - Unknown/Conflict
- Z - High-impedance/Open-circuit

Boolean Laws and Properties

Absorption Laws

- p|(p&q)≡p
- p&(p|q)≡p

Associative Laws (Associativity Property)

- (p|q)|r≡p|(q|r)
- (p&q)&r≡p&(q&r)

Biconditional

- ((p→q)&(q→p))≡(p↔q)

Boundedness Identity

- p&F≡F
- p|T≡T (*Annihilator*)
- p|F≡p

Commutative Laws (Commutativity Property)

- p|q≡q|p
- p&q≡q&p

Complementation

- ~T≡F and ~F≡T ∴ p|~p≡T

De Morgan's Laws

- ~(p&q)≡~p|~q
- ~(p|q)≡~p&~q

Distributive Laws (Distribution Property)

- p|(q&r)≡(p|q)&(p|r)
- p&(q|r)≡(p&q)|(p&r)

Domination Laws

- p|T≡T
- p&F≡F

Double Negation Law

- ~(~p)≡p

Exportation

- (p&q)→r≡p→(q→r)
- p↔q≡(p&q)|(~p&~q)

Idempotent Laws

- p|p≡p
- p&p≡p

Identity Property

- p&T≡p
- p|F≡p

Implication

- T→T≡T
- T→F≡F
- F→q≡T

Involution

- ~~p≡p

Negation Laws

- p|~p≡T
- p&~p≡F

Operator Precedence (Order)

- Parenthesis
- NOT
- AND
- OR and XOR
- IMPLICATION and EQUIVALENCE

Reflexivity

- p↔p

Substitution

- p|q≡~(~p&~q)
- p⊻q≡~(~(p&~(p&q))&~(q&~(p&q)))
- p⊻q≡~(~(p|q)|~(~(p|p)|~(q|q)))
- p⊻q≡(p|q)&(~(p&q))
- ~p≡~(p&p)
- ~p≡~(p|p)
- p&q≡~(~(p&q)&~(p&q))

Symmetry

- (p↔q)≡(q↔p)

Totality

- (p→q)|(q→p)

Transposition

- (p→q)≡(~q→~p) (*Contrapositive*)
- (p→q)≡(~p|q)

Transitivity

- ((p↔q)&(q↔r))→(p↔r)

Uniqueness of the Complement

- p|q≡T and p&q≡F → q=~p and p=~q

NOTES

- AND, OR, XOR, and EQUIVALENCE are commutative. All commutative operators are also associative.
- IEEE-1364 defines a four-valued logic system
- IEEE-1164 defines a nine-valued logic system

Classic Elements of Arguments

- Constructive Dilemma - ((p|q)&(p→r)&(q→s))→(r|s)
- Disjunctive Syllogism - ((p|q)&~p)→q
- Modus Ponens (Detachment) - (p&(p→q))→q
- Modus Tollens - (~q&(p→q))→~p
- Syllogism - ((p→q)&(q→r))→(p→r)

Made by Devyn Collier Johnson <DevynCJohnson@Gmail.com> (2015)